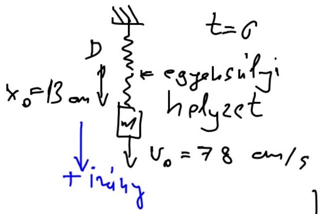
Fizika I 2. gyakorlat megoldások

F.1

$m = 2{,}5\,kg$
$D = 90\,N/m$



$$\omega = \sqrt{\frac{D}{m}} = 6\,\frac{1}{s}$$

a)

$x(t) = A\sin(\omega t + \varphi)$ (1)
$v(t) = \omega A\cos(\omega t + \varphi)$ (2)

$t = 0$: $x_0 = A\sin\varphi$, $v_0 = \omega A\cos\varphi$

$$tg\,\varphi = \frac{x_0}{v_0/\omega} = \frac{x_0\omega}{v_0} = \frac{6\frac{1}{s}\cdot 13\,cm}{78\,\frac{cm}{s}} = 1 \Rightarrow \varphi = 45^\circ = \frac{\pi}{4}$$

$x(t) = A\cdot\cos(\omega t + \varphi')$
$v(t) = -\omega A\sin(\omega t + \varphi')$

$t = 0$: $x_0 = A\cdot\cos\varphi'$, $v_0 = -\omega A\sin\varphi' > 0$

$$tg\,\varphi' = -\frac{v_0}{x_0\omega}$$

$$\varphi' = -45^\circ = -\frac{\pi}{4},\ \frac{3\pi}{4}$$

$$A^2 = x_0^2 + \frac{v_0^2}{\omega^2}$$

vagy (1)-ből

$$A = \frac{x_0}{\sin\varphi_0} \qquad A = \frac{x_0}{\cos\varphi'}$$

$$\boxed{A = \sqrt{2}\cdot 13\,cm = 18{,}4\,cm}$$

$$x(t) = 18{,}4\,cm\cdot\sin\left(6\frac{1}{s}\cdot t + \frac{\pi}{4}\right), \qquad x(t) = 18{,}4\,cm\cdot\cos\left(6\frac{1}{s}\cdot t + \frac{3\pi}{4}\right)$$

mikor megy át az egyensúlyi helyzeten? $(x = 0)$

$0 = \sin(6t + \frac{\pi}{4}) \Rightarrow$
$6t + \frac{\pi}{4} = n\cdot\pi$, $n = 1$
$n = 0 \Rightarrow t < 0$

$$\boxed{t_{min}} = \frac{\pi - \frac{\pi}{4}}{6} = \boxed{\frac{\pi}{8} \approx 0{,}40\,s}$$

$0 = \cos(6t + \frac{3\pi}{4}) \Rightarrow$
$6t + \frac{3\pi}{4} = (2n+1)\frac{\pi}{2}$, $n = 1, 2$

$$t_{min} = \frac{\frac{3\pi}{2} - \frac{3\pi}{4}}{6} = \frac{\frac{3}{4}\pi}{6} = \frac{\pi}{8}$$

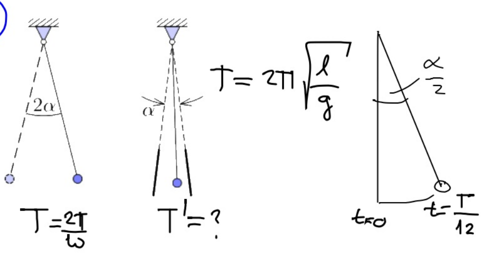
b)

$x_{max} = A = 18{,}4\,cm$
$v_{max} = \omega A = 110\,cm/s$
$a_{max} = \omega^2 A = 660\,\frac{cm}{s^2} = 6{,}6\,\frac{m}{s^2}$

c)

$$\boxed{E} = \frac{1}{2}DA^2 = \frac{1}{2}\cdot 90\,\frac{N}{m}\cdot(0{,}184\,m)^2 = \boxed{1{,}5\,J}$$

F2.



$$T = 2\pi\sqrt{\frac{l}{g}}$$

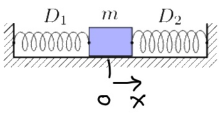
$\alpha(t) = \alpha\sin\omega t$

$\frac{\alpha}{2} = \alpha\cdot\sin\omega t$

$\sin\omega t = \frac{1}{2}$, $t = \frac{\pi}{6}\cdot\frac{1}{\omega} = \frac{2\pi}{\omega}\cdot\frac{1}{12} = \frac{T}{12}$

$$\boxed{T = 4\cdot t = \frac{T}{3}}$$

F3

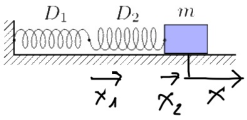


$$F = -D\cdot x = -(D_1+D_2)\,x$$

$$\omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{D_1+D_2}{m}}$$

$F_1 = D_1 x \qquad F_2 = D_2 x$

$$T = \frac{2\pi}{\omega} \qquad \boxed{T = 2\pi\sqrt{\frac{m}{D_1+D_2}}}$$



$$x = x_1 + x_2$$

$F = -D x$

$F = -D_1 x_1 = -D_2 x_2$

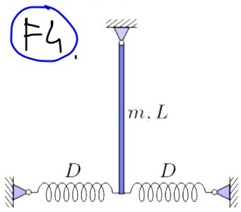
$x = \frac{F}{D},\ x_1 = \frac{F}{D_1},\ x_2 = \frac{F}{D_2}$

$$\frac{1\cancel{F}}{D} = \frac{1\cancel{F}}{D_1} + \frac{1\cancel{F}}{D_2} \Rightarrow \frac{1}{D} = \frac{1}{D_1} + \frac{1}{D_2}$$

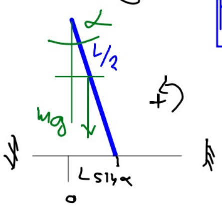
$$\omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{D_1 D_2}{m(D_1+D_2)}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m(D_1+D_2)}{D_1 D_2}}}$$

F4.



fizikai inga



$\boxed{M = \Theta\beta}$, ahol $\beta = \ddot{\alpha}$

$$M = -mg\cdot\frac{L}{2}\sin\alpha - \underbrace{DL\sin\alpha}_{\text{bal rugó nyúlik}} - \underbrace{DL\sin\alpha}_{\text{jobb rugó összenyomódik}}$$

$$\Theta = \frac{1}{3} m L^2$$

$$\Downarrow$$

$$-mg\cdot\frac{L}{2}\sin\alpha - 2DL\sin\alpha = \frac{1}{3} m L^2\,\ddot{\alpha}$$

$$-\left(\frac{mg}{2} + 2D\right) L\sin\alpha = \frac{1}{3} m L^2\cdot\ddot{\alpha} \qquad (*)$$

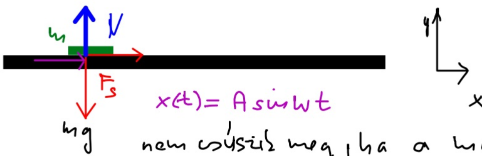
$$\boxed{\ddot{\alpha} = -\frac{(mg+4D)\cdot 3}{2mL^2}\sin\alpha}$$

ha α kicsi $(\alpha << 1)$ $\Rightarrow \sin\alpha \approx \alpha$

$\ddot{\alpha} = -\frac{(mg+4D)\cdot 3}{2mL^2}\alpha$, vagyis $\ddot{\alpha} = -\omega^2\alpha$ alakú

ahol $\boxed{\omega = \sqrt{3\frac{mg+4D}{2mL^2}}}$ $\boxed{\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{3\frac{mg+4D}{2mL^2}}}$

15B–5



$x(t) = A\sin\omega t$

nem csúszik meg, ha a max. gyorsulásnál sem csúszik meg, azaz

$mg - N = 0$

$N = mg$

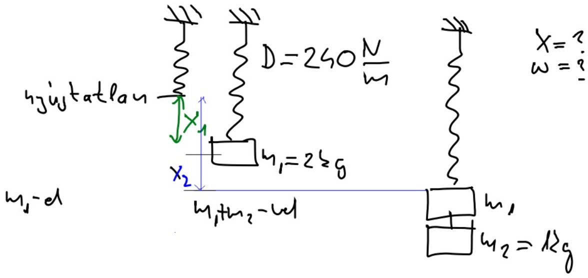
$F_s = m \cdot a_{max} = m\omega^2 A$

$F_s \le \mu \cdot N \qquad \mu_{min} \Rightarrow F_s = \mu m g$

$\mu \cancel{m} g = \cancel{m}\omega^2 A$

$$\boxed{\mu = \frac{\omega^2 A}{g}}$$

15B – 13



$x_1 = \frac{m_1 g}{D} = \frac{20}{240} = \frac{1}{12}\,m$

$x_2 = \frac{m_1+m_2}{D}g = \frac{30}{240} = \frac{1}{8}\,m$

x_2 az egyensúlyi helyzet $\Rightarrow$ hozzá képest

$t=0 \Rightarrow x(0) = -(x_2-x_1)$

$v(0) = 0$

$x_2 - x_1 = \frac{1}{8} - \frac{1}{12} = \frac{4}{96} = \frac{1}{24}\,m$

$A\sin\varphi = -(x_2-x_1)$

$\omega A\cos\varphi = 0 \qquad \Rightarrow \varphi = \frac{\pi}{2} + n\cdot\pi$

$\sin\varphi = -1 \Rightarrow \varphi = \frac{3\pi}{2}$

$A = (x_2 - x_1) = \frac{1}{24}\,m = 0{,}0417\,m$

a) max kitérés $= 2A \boxed{= 0{,}0834\,m}$

$(m_1+m_2)a = -Dx^2$

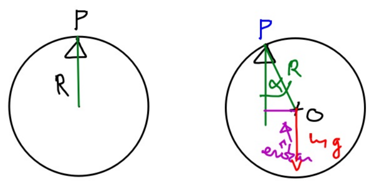
$\ddot{x} = -\frac{D}{m_1+m_2}x$

$x(t) = A\sin(\omega t + \varphi)$

$\omega = \sqrt{\frac{D}{m_1+m_2}} = \sqrt{\frac{240}{3}} = \sqrt{80}\,\frac{1}{s} = 8{,}94\,\frac{1}{s}$

$\nu = \frac{\omega}{2\pi} = 1{,}42\,\frac{1}{s}$

15B-26



$M_p = \Theta_p \beta$

$\Theta_p = \Theta_0 + mR^2$ (Steiner tétel)

$\Theta_0 = mR^2 \qquad \Theta_p = 2mR^2$

a) $M_p = mgR\sin\alpha \Rightarrow -\cancel{m}g\cancel{R}\sin\alpha = 2\cancel{m}R^{\cancel{2}}\cdot\ddot{\alpha}$

$$\ddot{\alpha} = -\frac{g\sin\alpha}{2R} \approx -\frac{g}{2R}\cdot\alpha \quad \text{ha } \alpha \ll 1$$

formailag ez is harmonikus rezgőmozgás

$$\ddot{\alpha} \approx -\omega^2 x \qquad \omega = \sqrt{\frac{g}{2R}} = \sqrt{\frac{10\,m/s^2}{2\cdot 0{,}2\,m}} = 5\,\frac{1}{s}$$

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{5}\,s = 1{,}26\,s}$$

b) matematikai inga: $\omega = \sqrt{\frac{g}{L}} \Rightarrow \boxed{L = 2R}$

HN15B-28 a csillapított harmonikus rezgőmozgás egyenlete

$$m\ddot{x} = -Dx - k\dot{x} \qquad k \text{ – csillapítási együttható}$$

átírva

$$\ddot{x} = -\frac{D}{m}x - \frac{k}{m}\dot{x} \qquad \omega_0 = \sqrt{\frac{D}{m}} \qquad \beta = \frac{k}{2m}$$

$$\ddot{x} = -\omega_0^2 x - 2\beta\dot{x}$$

megoldása:

$$x(t) = A(t)\sin(\omega t + \varphi) \quad \text{vagy u.e. cos-al}$$

a) ismert: $A(0) = 0{,}20\,m \quad A(6s) = 0{,}16\,m$

$$A(t) = A(0)e^{-\beta t}$$

$$\ln\frac{A(t)}{A(0)} = -\beta t$$

$$t = 6s \Rightarrow \ln\frac{0{,}16}{0{,}20} = -\beta\cdot 6s \Rightarrow -0{,}223 = -6s\,\beta$$

$$\beta = 0{,}037\,\frac{1}{s}$$

$$\boxed{k = 2m\beta \doteq 0{,}1488\,kg/s}$$

b) rezonancia frekvencia:

$$\omega_r = \sqrt{\omega_0^2 - 2\beta^2} \approx \omega_0 \approx \boxed{10\,\frac{1}{s}}$$

$$\left(\frac{\beta}{\omega_0} \ll 1\right)$$